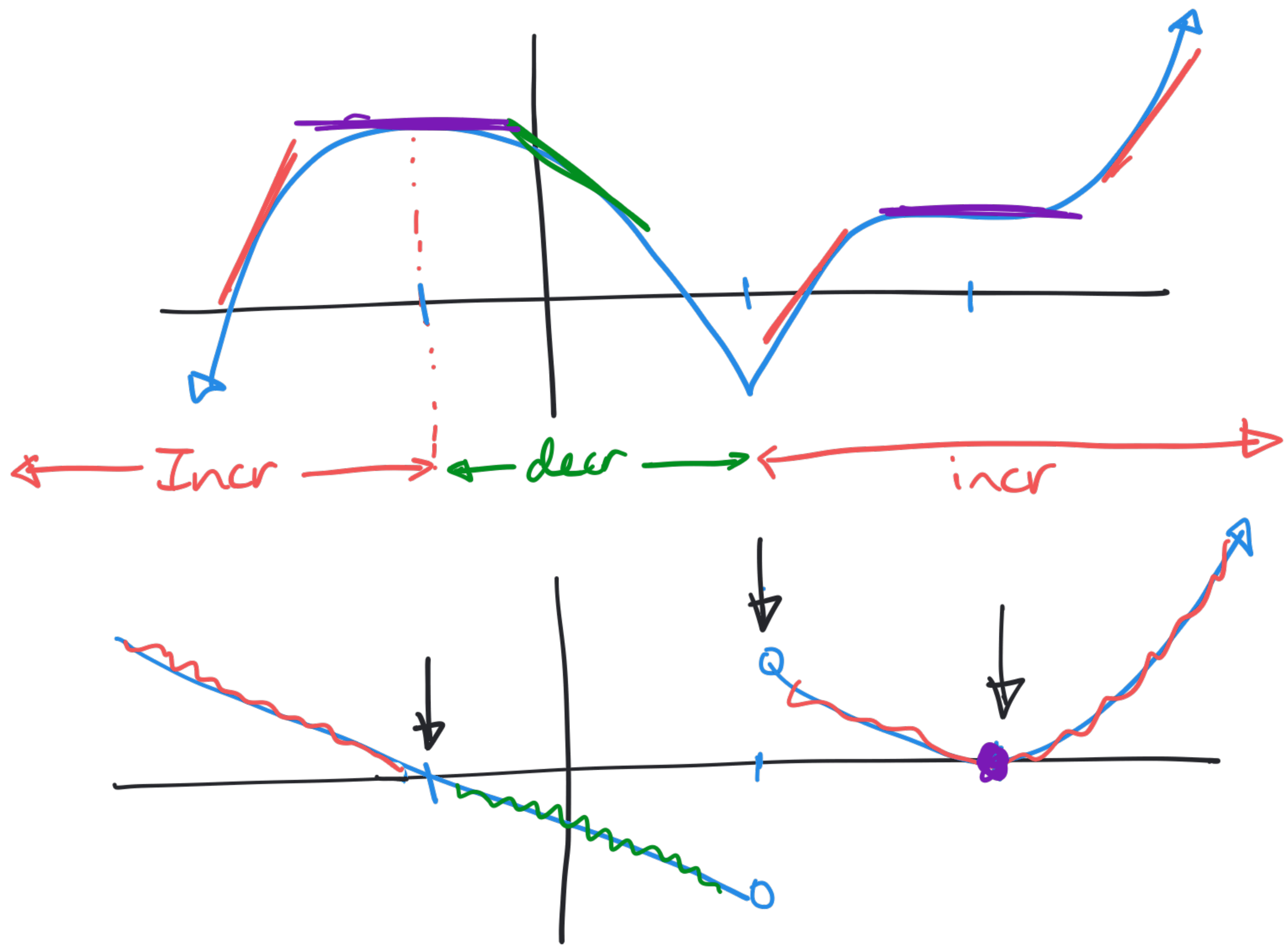
Intro Video: Section 4.3 How derivatives affect the shape of a graph

Math F251X: Calculus I

Increasing/Decreasing

- Derivative > 0 $\iff$ function $\nearrow$
- Derivative < 0 $\iff$ function $\searrow$



Example: $f(x) = \frac{3}{2}x^4 - 9x^2 - 12x + 1$

Where is f increasing? Where is f decreasing?

FACT: f increasing $\Rightarrow f' > 0$

f decreasing $\Rightarrow f' < 0$

the only place f' can change sign is at a critical point!

$$f'(x) = \frac{3}{2}(4x^3) - 9(2x) - 12 = 6x^3 - 18x - 12$$

$$= 6(x^3 - 3x - 2)$$

$$= 6(x+2)(x^2 - 2x - 1)$$

$$= 6(x+2)(x-1)^2$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & -2 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

(No kidding, it's synthetic division. You do NOT need to know how to do this, although polynomial division is helpful.)

$$f'(x) = 0 \Rightarrow 6(x+2)(x-1)^2 = 0$$

$$\Rightarrow \boxed{x = -2 \text{ or } x = 1}$$

$f'(x)$ undefined? Nowhere!

$f(x) = \frac{3}{2}x^4 - 9x^2 - 12x + 1$

$f'(x) = 6(x+2)(x-1)^2$

Critical points are $x = -2$, $x = 1$

x	$x < -2$	-2	$-2 < x < 1$	1	$x > 1$
Sample	-3		0		2
Sign of f'	$-$	0	$+$	0	$+$
behavior of f	↘	$-$	↗	$-$	↗

$f'(-3) =$
$= 6(\underbrace{-3+2}_{\downarrow})(\underbrace{-3-1}_{\downarrow})^2$
$= 6(-)(-)^2$
$= +(-)(+)$

$f'(2) = 6(2+2)(2-1)^2$
$= 6(+)(+)^2$
$= +$

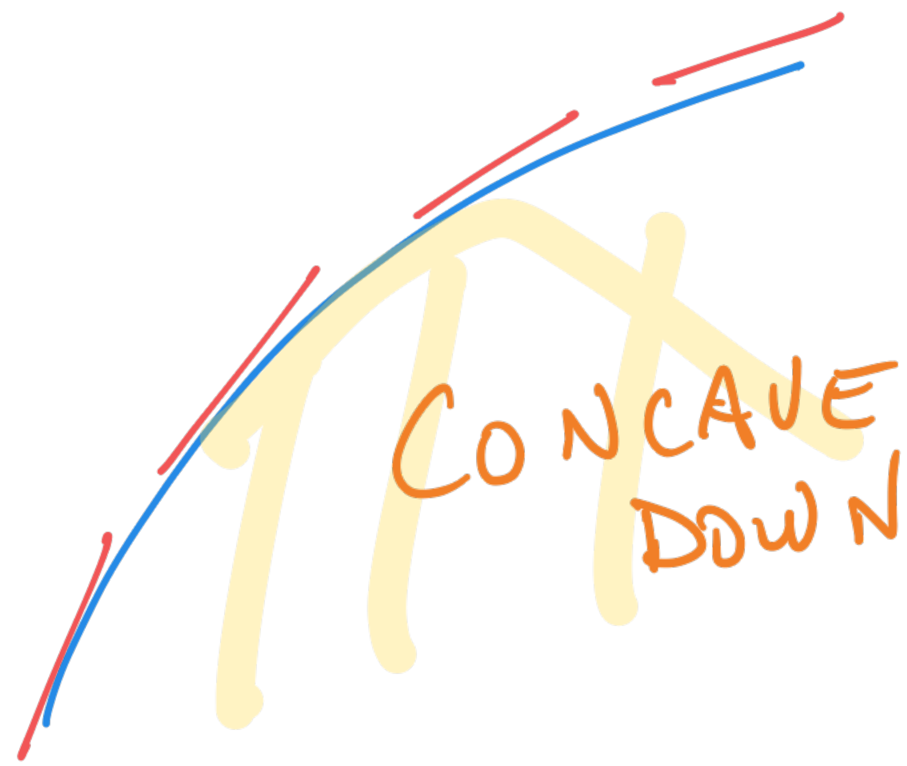
$f'(0) = 6(2)(0-1)^2$
$= +$

Intervals of increase: $(-2, 1) \cup (1, \infty)$

Intervals of decrease: $(-\infty, -2)$

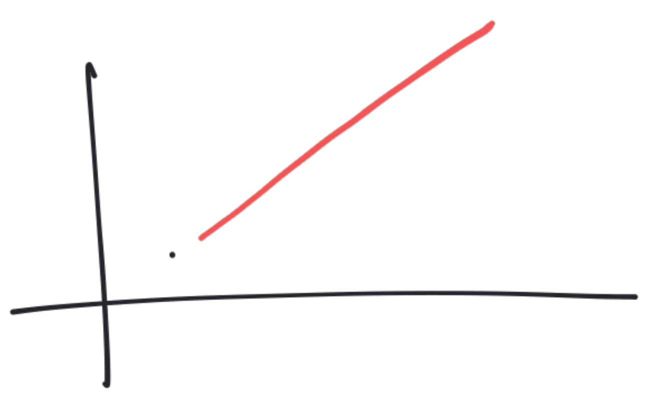
How do functions increase or decrease?

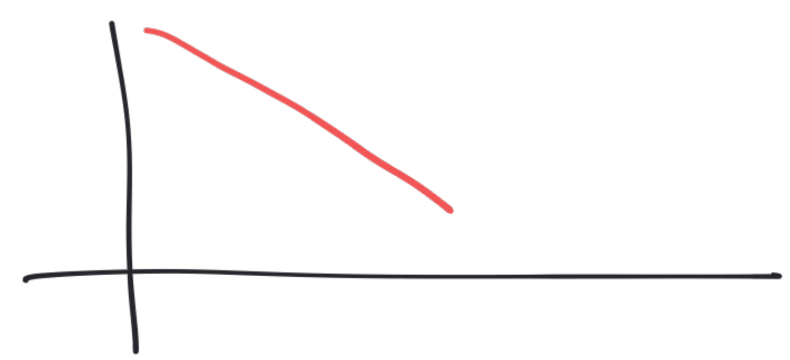
Slopes are getting steeper



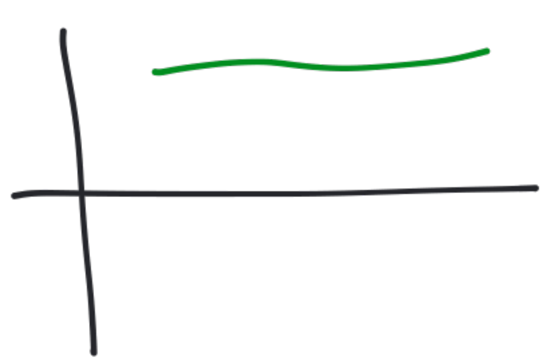
positive slope, but getting ~~less~~ steep

f'





f''



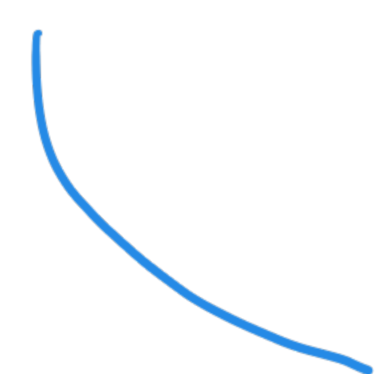
A function f is

CONCAVE UP

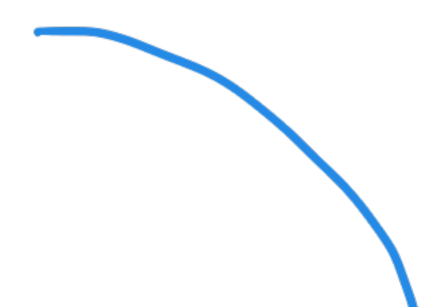
if $f''(x) > 0$

CONCAVE DOWN

if $f''(x) < 0$



Decreasing and CU



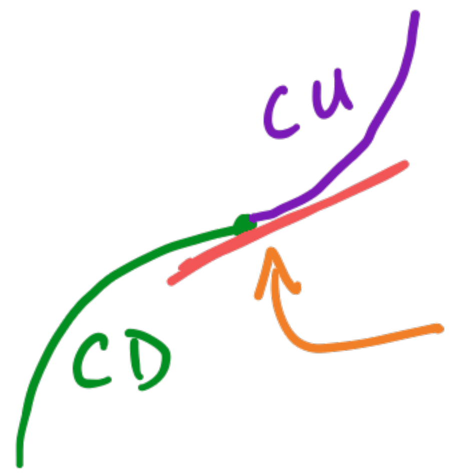
Decreasing and CD

Example: $f(x) = \frac{3}{2}x^4 - 9x^2 - 12x - 1$

$$f'(x) = 6x^3 - 18x - 12 = 6(x-2)(x+1)^2$$

critical pts at $x=2$, $x=-1$

$$f''(x) = 6(3x^2) - 18 = 18x^2 - 18 = 18(x-1)(x+1)$$



Inflection point is where a function changes concavity

Where is $f(x)$ CU? CD? Where does it have inflection points?

$\rightarrow$ find critical points for <u>first derivative</u> to see where f might change concavity!

Find where $f''(x) = 0 \Rightarrow 18(x-1)(x+1) = 0 \Rightarrow x = 1,\ x = -1$

(note $f''(x)$ is never undefined)

f'	$+$	$-$	$-$	$+$
f''	$-$	$-$	$+$	$+$
	CD		CU	

Recall: $f(x) = \frac{3}{2}x^4 - 9x^2 - 12x + 1$, $f''(x) = 18(x-1)(x+1)$

x	$x < -1$	-1	$-1 < x < 1$	1	$x > 1$
Sample	-2		0		2
Sign of f''	$+$	0	$-$	0	$+$
CU/CD					

inflection points

$$f''(-2) = 18(-2-1)(-2+1) = 18(-)(-) = +$$

$$f''(0) = 18(0-1)(0+1) = 18(-)(+) = -$$

$$f''(2) = 18(2-1)(2+1) = 18(+)(+)$$

What do we know about $f(x) = \frac{3}{2}x^4 - 9x^2 - 12x + 1$?

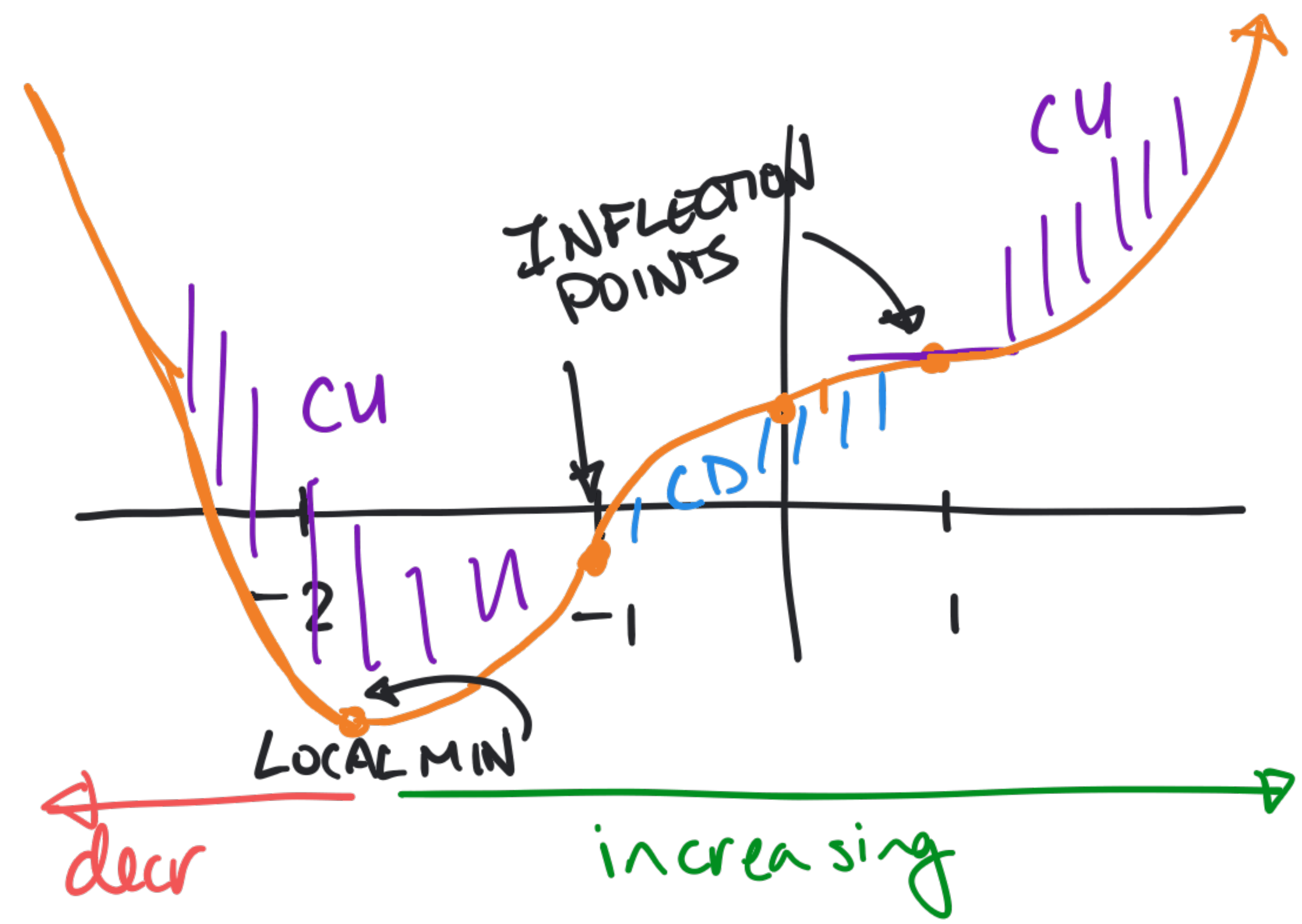
incr/decr

x	$x < -2$	-2	$-2 < x < 1$	1	$x > 1$
f'	$-$	0	$+$	0	$+$
f	↘		↗		↗

CU/CD

x	$x < -1$	-1	$-1 < x < 1$	1	$x > 1$
f''	$+$	0	$-$	0	$+$
f	∪		∩		∪

x	$x < -2$	-2	$-2<x<-1$	-1	$-1 < x < 1$	1	$x > 1$
f'	$-$		$+$		$+$		$+$
f''	$+$		$+$		$-$		$+$
↗	↘	∪	↗	~	↗	~	↗
∪							



-2 is a local min

-1 and 1 are both inflection points, but at $+1$ has a flat tangent line whereas at $x = -1$ the TL has positive slope